RESEARCH STATEMENT

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My research lies at the intersection of geometric group theory, algebraic topology and coarse geometry. I am interested in how algebraic and geometric structures of groups are reflected in their large-scale and homological invariants. Much of my current work focuses on understanding large-scale analogs of classical invariants like Lusternik-Schnirelmann Category, Cohomological dimension, etc. I am also interested in persistent homology and cohomology of spaces, when the space is endowed with a group action.

1. Completed Research

1.1. **Coarse Lusternik-Schnirelmann Category.** My most recent work is in coarse geometry [4]. I define a coarse version of the classical Lusternik-Schnirelmann (LS) Category of a space X, denoted by c-cat(X). This is a continuation of work by R. Ayala and co-authors, who introduced a similar notion in the proper homotopy category, called 'Proper LS-Category' p-cat(X).

In my most recent paper I show that c-cat(X) is an invariant up to coarse homotopy equivalence, and give some upper and lower bounds for it.

- For geometrically finite groups Γ, we show the inequality
$$\text{p-cat}(\Gamma) \leq \text{c-cat}(\Gamma).$$
Since there are known cohomological lower-bounds for p-cat(), this gives us good lower bounds for computation purposes.
- We show that for bicombable 1-ended groups which are semistable at ∞, we have an upper bound:
$$\text{c-cat}(\Gamma) \leq \text{asdim}(\Gamma).$$
This is a nice analog of the classical result that states that $\text{cat}(X) \leq \dim X$, where cat() stands for the (reduced) classical LS-category, and dim is the covering dimension.

1.2. **Cohomological Dimension of Group Homomorphisms.** Recall that for a group G, the cohomological dimension $\text{cd}(G)$ is defined as the largest n such that $H^i(G, M) \neq 0$ for some G-module M. The homological dimension $\text{hd}(G)$ is defined the same way, replacing cohomology with homology. Extending those definitions, one can define for a group homomorphism $\varphi : G \to H$:

$$\text{cd}(\varphi) = \max\{n \mid \varphi^* : H^n(H, M) \to H^n(G, M) \text{ is nontrivial for some } G\text{-module } M\}$$

$$\text{hd}(\varphi) = \max\{n \mid \varphi^* : H_n(G, M) \to H_n(H, M) \text{ is nontrivial for some } G\text{-module } M\}$$

In my joint paper with Alexander Dranishnikov [2], we prove several properties of $\text{cd}(\varphi)$:

- $\mathrm{cd}(\varphi) = \mathrm{hd}(\varphi)$. The cohomological, homological dimension of the homomorphisms coincide, whenever both groups G, H are geometrically finite. This is a generalization of the classical result which says that for geometrically finite groups G, homological and cohomological dimensions coincide.
- We prove $\mathrm{cd}(\varphi \times \varphi) = 2\mathrm{cd}(\varphi)$ for φ between geometrically finite groups. This is a generalization of a result by Dranishnikov 2019, where he proved the square law for geometrically finite groups.
- We give examples of group homomorphisms φ for which $\mathrm{cd}(\varphi) < k$ but $\mathrm{gd}(\varphi) = k$. This shows that we cannot have analogs for the classical Eilenberg-Ganea theorem.

1.3. **Persistent Equivariant Cohomology.** In our paper [1] (Joint work with H. Adams, E. Lagoda, M. Moy, N. Sadovek) We define the notion of *Persistent Equivariant Homology* (and Cohomology) where we consider the category of filtered simplicial complexes, which are equivariant under the action of a topological group. The particular action we considered in this paper was the (natural) action of the circle on the (metric thickening) Vietoris-Rips complex of S^1, which we'll denote by $\mathrm{VR}^m(S^1; r)$. Here r denotes the scale.

- It is known that $\mathrm{VR}^m(S^1; r) = S^{2k+1}$ up to homotopy, where the dimension of the sphere $2k+1$ depends on the choice of scale r. We gave a proof of this fact which is S^1-equivariant, and the S^1-action on the $2k+1$ odd-sphere is not obvious. We gave a description of the action in the paper as well.
- We computed the S^1-equivariant persistent cohomology of $\mathrm{VR}^m(S^1; r)$ using the previous equivalence.

This research opens new avenues for analyzing data with inherent symmetries or group actions.

1.4. **Boundary of Right-angled Coxeter Groups.** It is an open conjecture which states that the boundary of any word-hyperbolic group admits a fixed-point-free involution. In a short note [3], I show that such a statement is true for right-angled coxeter groups. This doesn't solve the conjecture completely (although there are reasonable families of RACGs which are hyperbolic), however the techniques may prove useful for possible approaches to the problem.

2. Future Directions

2.1. **Coarse LS Category.** There are quite a few exciting open questions in this area.

- There are quite a few families of groups whose asymptotic dimension is not known to be finite (For example, Helly Groups, groups with higher hyperbolicity etc.). It would be interesting to see if we can compute c-cat() for these groups instead.
- Because of the inequality $\text{p-cat}(X) \leq \text{c-cat}(X)$, we can use the cohomological obstructions (for example, cup-length in cohomology theory with compact supports) as lower bounds for c-cat(), as they are lower bounds for p-cat(). We would like some cohomological obstructions in the coarse cohomology theory due to Roe. Unfortunately the cup product in Roe's coarse cohomology HX is zero, so he defined a secondary product structure on the coarse cohomology groups. This new product structure is quite strange, but could tell us a lot about HX, and how it relates to c-cat().

- Ganea and Whitehead had different constructions for LS-category, it would be interesting to see if we can have similar constructions for coarse LS-category, in the coarse setting.
- Classically we know that $\mathrm{cat}(X) \leq \dim \partial X + 1$ for hyperbolic spaces X, with Gromov's visual boundary ∂X. We would like to see similar results for coarse category, where instead of considering hyperbolic spaces and their visual boundaries, we consider any space which admits a Z-boundary. In particular, we conjecture that $\text{c-cat}(X) \leq \dim \partial X + 1$.

2.2. **Cohomological Dimension of Group Homomorphisms.** There is a known notion of cohomological dimension of a group G relative to a family of subgroups $\mathcal{F}$ of G, denoted by $\mathrm{cd}_{\mathcal{F}}(G)$. It is an interesting question to ask, for surjective homomorphism $\varphi : G \to H$, do we have the inequality $\mathrm{cd}(\varphi) = \mathrm{cd}_{\mathcal{F}}(G)$, where $\mathcal{F}$ is the family of subgroups generated by the normal subgroup $\ker \varphi$? This would allow us to rephrase a lot of our results in terms of relative cohomological dimension.

References

[1] Henry Adams et al. *Persistent Equivariant Cohomology*. Aug. 30, 2024. DOI: 10.48550/arXiv.2408.17331. arXiv: 2408.17331. URL: http://arxiv.org/abs/2408.17331.

[2] Aditya De Saha and Alexander Dranishnikov. "On Cohomological Dimension of Homomorphisms". In: *Proceedings of the American Mathematical Society* 152.11 (Nov. 2024), pp. 4607–4621. ISSN: 0002-9939, 1088-6826. DOI: 10.1090/proc/16943. URL: https://www.ams.org/proc/2024-152-11/S0002-9939-2024-16943-4/.

[3] Aditya De Saha. *Fixed-Point-Free Involutions on the Boundaries of Right-angled Coxeter Groups*. Mar. 14, 2025. DOI: 10.48550/arXiv.2503.11725. arXiv: 2503.11725 [math].

[4] Aditya De Saha. *On the Coarse Lusternik-Schnirelmann Category of Groups*. Nov. 2025. DOI: 10.48550/arXiv.2510.10367. eprint: 2510.10367 (math).